Engineering Notes

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Probability of Crashing from Monte Carlo Simulation

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Introduction

MANY terrain-following studies rely heavily on Monte Carlo simulation to obtain an estimate of the probability of crashing or "clobber" (P_c) for a missile flying in terrain-following mode over various types of terrain. This estimate for P_c is then used to adjust the command clearance setting. In some simulations, somewhat arbitrary random effects are introduced such as wind gusts and obstacles. In others, a long enough stretch of terrain provides the random effect desired.

With a reasonably complete missile/autopilot simulation, these simulated flights over long stretches of terrain become very expensive. Two related questions need to be answered.

1) Do the results justify the expense? In other words, what confidence can one place in the estimate of P_c obtained?

2) How long a stretch of terrain or how many independent Monte Carlo trials are needed before the estimate of P_c is close to the true value?

A method to answer these questions is developed in this Note.

Estimating Probability of Crashing from Monte Carlo Simulation

As shown in Ref. 1, the probability of crashing for a flight length T is

$$P_c(T,h_0) = 1 - \exp[-\lambda(h_0)T] \tag{1}$$

where $\lambda(h_0)$ is the mean frequency of crashing (crashes/s or crashes/n. mi., depending upon the units of T the missile speed being approximately constant) and is a function of missile altitude clearance command h_0.

An estimate $\hat{\lambda}$ of λ can be obtained from a series of Monte Carlo simulation runs with fixed h_0, as

$$\hat{\lambda}(h_0) = n_c(h_0)/T_{mc} \tag{2}$$

where T_{mc} is the combined length of the runs and n_c is the number of crashes observed. Omitting the h_0 dependence for brevity, a estimate of P_c is based on these Monte Carlo runs is given by

$$\hat{P}_c(T) = 1 - \exp[-(n_c/T_{mc})T] \tag{3}$$

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Note from Eq. (3) that if even one crash is counted in T_{mc}, the estimate P_c over the length T_{mc} will be 63.2%, and with two crashes 86.5%, etc. This implies a much shorter length T is needed to achieve a P_c on the order of, say, 3%. From Eq. (3) with $n_c = 1$ and $T \ll T_{mc}$, $T_{mc} \approx T/\hat{P}_c(T)$. For example a P_c estimate of 3%, over 500 miles would require a total Monte Carlo length of 500/0.03 = 16,667 miles with only one crash observed in that length. Similarly one crash in $T_{mc} = 1000$ miles would imply a P_c of 3% in 30 miles. The estimate of P_c given by Eq. (3) is plotted for values of n_c in Fig. 1.

It is evident that Monte Carlo simulation to estimate *small* P_c values for reasonable lengths of terrain would rapidly become prohibitively expensive, especially if a relatively complete missile/autopilot simulation is included. As an alternative method for estimating P_c, the total length T_{mc} could be divided into n_{mc} independent intervals each of length L_c where L_c is the correlation length of the terrain. Thus, $T_{mc} = n_{mc}L_c$. The intervals are statistically independent in the sense that a crash in one of the interval does not affect the prohibility of a crash in any other interval. Also, it is assumed that there is no possibility of there being more than one crash in any one interval. Note that these assumptions are in line with those underlying the Poisson expression for P_c given in Eq. (1), which requires that the events of a crash be statistically independent.

Suppose that p is the true probability of crashing in any one of the intervals. Then the probability of crashing in n such intervals or trials is

$$P_c(n) = 1 - (1-p)^n \tag{4}$$

If n_c crashes are counted in n_{mc} trials then an estimate of p is given by

$$\hat{p} = n_c/n_{mc} \tag{5}$$

and the resulting Monte Carlo estimate for P_c is

$$\hat{P}_c(n) = 1 - (1 - n_c/n_{mc})^n \tag{6}$$

As $n_{mc} \to \infty$ and $L_c \to 0$, T_{mc} remaining fixed, $\hat{P}_c$ given by Eq. (6) approaches that given by Eq. (3).

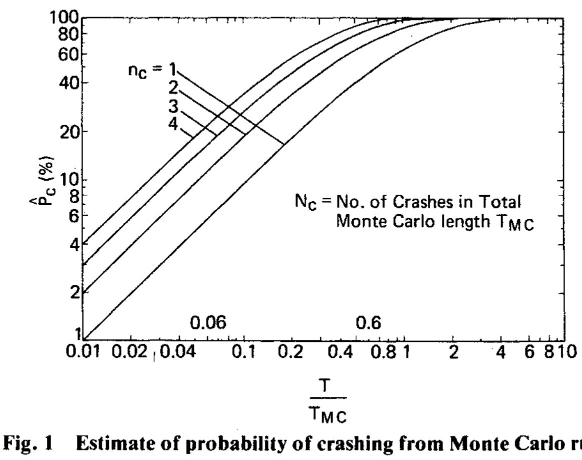

Fig. 1 Estimate of probability of crashing from Monte Carlo runs.

Confidence Limits for the Probability of Crashing

Having estimated the probability of crashing according to Eq. (3) or Eq. (6) from a Monte Carlo simulation, the question arises – how much trust can be placed in such an estimate? Perhaps the easiest approach to obtaining confidence limits for P_c is as follows. First obtain confidence limits for p, the true probability of crashing in *one* independent trial and then extrapolate these limits to P_c by means of Eq. (4).

Suppose a finite number n of independent intervals is considered and the number n_c in which a crash occurred is counted. Then an estimate of p, based on this set of n runs, is given by Eq. (5) with n_{mc} replaced by n, the latter symbol being used for brevity here. The designation n_{mc} will be used when the distinction between n and n_{mc} is necessary. Now if another set of n runs (n fixed) is taken, another value for n_c would probably result. Hence, in advance of experimentation, the number of runs in which a crash occurred may be treated as a random variable N_c.

The sampling distribution for N_c is binominal, i.e.,

$$P\{n_1 \leq N_c \leq n_2\} = \sum_{k=n_1}^{n_2} \binom{n}{k} p^k q^{n-k} \quad (7)$$

where n_1 and n_2 are integers and $q = 1-p$. Define a new random variable $Z = (N_c - np)/\sqrt{npq}$. This standardized random variable has zero mean and a variance of unity. For $n > 30$ and $0.03 \leq p \leq 0.97$, the Gaussian distribution is a good approximation to the binomial distribution. Accordingly, under these conditions, Eq. (7) may be written as

$$P\left\{-z_{\alpha/2} \leq \frac{N_c - np}{\sqrt{npq}} \leq z_{\alpha/2}\right\} = 1 - \alpha \quad (8)$$

where $\alpha/2 = P\{Z > z_{\alpha/2}\}$. For example, if $\alpha = 0.1$, then, from Gaussian density function tables, $z_{\alpha/2} = 1.645$. Thus, if p were known, then *in advance* of carrying out n trials it could be stated that there is a 90% probability that the number of crashes which will occur lies between $np + 1.645\sqrt{npq}$ and $np - 1.645\sqrt{npq}$. But p is not known. Instead $N_c = n_c$ is measured in the n Monte Carlo trials, and based on that measurement a confidence statement about the true probability p can be made. The interval within the bracket in Eq. (8), with N_c replaced by the measured number of crashes n_c, defines the $(1-\alpha)$ confidence interval for p. Solving for p gives the interval as $p_1 \leq p \leq p_2$ where, defining $A = z_{\alpha/2}$,

$$p_1, p_2 = \frac{2\,n_c + A^2}{2(n + A^2)} \pm \frac{1}{2(n+A^2)}\sqrt{(2n_c + A^2)^2 - \frac{4n_c^2}{n}(n + A^2)} \quad (9)$$

Table 1 **Comparison of 95% confidence limits for p using binomial distribution and Gaussian approximation**

No. of crashes[a] n_c	95% confidence limits Table 14.57.1 (Ref. 2)		Gaussian approximation (Eq. 9)	
0	.00	.04	.000	.037
1	.00	.05	.002	.054
2	.00	.07	.006	.070
3	.01	.08	.010	.084

[a] Number of Monte Carlo trials $n_{Mc} = 100$.

Using Eq. (9), 90% confidence limits for p are plotted on Fig. 2 for various n_{mc} for the case where a $\hat{p}(=n_c/n_{mc})$ of 3% is measured. It will be noted that with 100 independent lengths, one can say with 90% confidence that the true value of p lies between 1% and 7%, which is not very satisfactory. It is not difficult to show analytically that about 2700 independent lengths are needed before the 90% confidence limits for p lie within 20% of the measured $\hat{p}$ value.

The results of Fig. 2 for 100 Monte Carlo runs were used to get the curves on Fig. 3. This shows the overall probability of crashing estimate $\hat{P}_c$ for various n, based on Eq. (4) with p replaced by $\hat{p}$. The confidence limit curves are also based on Eq. (4) with p replaced by p_1 and p_2 as given in Eq. (9). It is evident that with an estimated 3% probability of crashing in one interval, not many intervals are required before $\hat{P}_c$ becomes large.

It was mentioned earlier that the Gaussian approximation to the binomial distribution was good for $n_{mc} > 30$ and $0.03 \leq p \leq 0.97$. The approximation is also good for smaller values of p provided n_{mc} is large enough (say $n_{mc} \geq 100$). For example, Table 1 gives a comparison between 95% confidence limits computed using incomplete beta functions for evaluating the binomial distribution, and limits obtained from Eq. (9).

The correspondence is seen to be quite good. It will be noted from the table that even if no crashes are observed, implying $\hat{p} = 0$, the most one can say with 95% confidence is that the true value p lies between 0 and 4%. This once again demonstrates the difficulty of obtaining accurate estimates for p from moderate numbers of Monte Carlo trials. For $n_{mc} = 1000$ with $n_c = 0$, Eq. (9) gives the 95% confidence interval for p as 0 and 0.38%. The nonzero upper limit becomes significant on extrapolation to get the overall probability of crashing in a length equivalent to n independent intervals. For example, using Eq. (4), the true P_c could lie with 95% confidence between 0 and 18% for $n = 50$ and between 0 and 54% for $n = 200$, etc., while the Monte Carlo estimate $\hat{P}_c$ is zero.

It is clear that one can put little trust in zero estimates of $\hat{P}_c$ even from large numbers of Monte Carlo trials. This leads to the suspicion that small nonzero values of p estimated from a

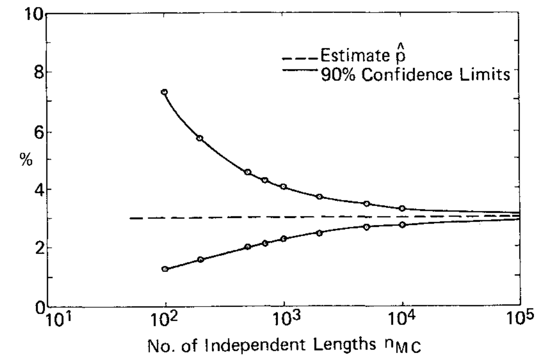

Fig. 2 90% confidence limits for p if 3% is measured.

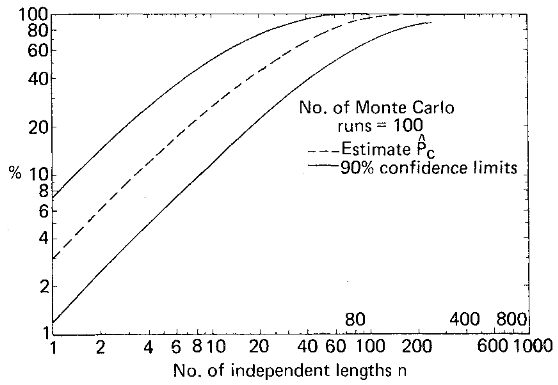

Fig. 3 90% confidence limits for P_c from 100 Monte Carlo runs.

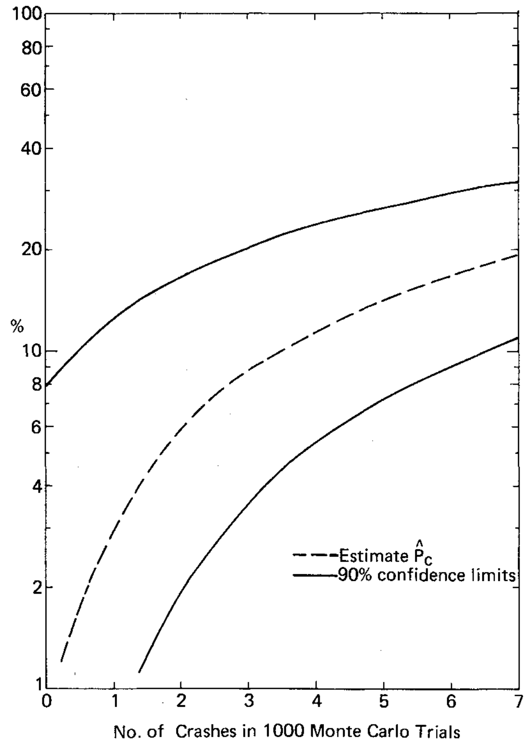

Fig. 4 90% confidence limits for P_c in 30 independent intervals based on 1000 Monte Carlo trials.

large number of trials also lead to unreliable estimates of $\hat{P}_c$ when extrapolated. This suspicion is confirmed for example with, say one crash observed in 1000 Monte Carlo trials giving $\hat{p} = 0.1\%$ with 90% confidence limits of 0.02% and 0.45%. When extrapolated to obtain the probability of crashing for 30 trials the estimate $\hat{P}_c$ is 3% while the 90% confidence limits are 0.7% and 12.6%. These results are included in Fig. 4, which gives confidence limits for various numbers of observed crashes. It is concluded that, even with a large number of trials, Monte Carlo methods are unreliable for estimating small values of probability of crashing.

Summary

Confidence limits are obtained for the probability of crashing based on the estimate provided by Monte Carlo simulation. This involves extrapolating the confidence limits for the probability of crashing for a single independent interval of terrain, to the desired terrain length. The method indicates how many independent intervals are needed in a given case before P_c and the estimate $\hat{P}_c$ are close.

Conclusions

1) Even with large numbers of trials, Monte Carlo methods are unreliable for estimating *small* values of P_c. Some other method, for example as suggested in Ref. 1, should be used.

2) With a sufficient number of trials, large values of P_c may be estimated with reasonable accuracy. However such large P_c values would automatically indicate that higher clearances be flown to reduce P_c and so one is still faced with the problem of a reliable estimate of the new smaller P_c values.

References

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Development of a Multipurpose Radiometer

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Introduction

THE early programs conducted in the large thermal-vacuum chambers at the Johnson Space Center (JSC) (circa 1966) involved measurement of thermal irradiance in the solar spectral region (0.25 to 2μ). A split-disc bolometer was used on these tests because it was the only commerically available instrument that had a flat spectral response over the solar spectral region and could operate within the thermal-vacuum environment.

As thermal-vacuum testing evolved in the ensuing years, it became necessary to measure irradiance in the infrared spectral region (0.8 to 20μ). The split-disc bolometer was not suitable in that its window would not transmit energy at wavelengths longer than 2μ. A Boelter-Schmidt-type thermopile with a black matte finish on the sensor surface was chosen for use in the infrared spectral region.

The two instruments coexisted for many years, each performing its own function. Eventually, technical and economic demands created a requirement for a single instrument that could be utilized in both spectral regions and could withstand the thermal-vacuum environment.

As a result, a program was initiated to develop such an instrument. The product of this development program is a hybrid thermal-type sensor. It incorporates the split-disc, windowed design of the bolometer to allow it to perform in the solar spectral region, but is easily convertible to infrared usage by removal of the window.

Description of Radiometer

The instrument consists of two semicircular Boelter-Schmidt-type sensors, i.e., split-disc. The sensors are copper constantan wire wound around an electrically insulative core with one-half of the winding copper plated (Fig. 1). The sensors are mounted adjacent to each other on a cylindrical aluminum base and painted with a black matte finish. The output leads from the sensors and a thermocouple extends from the side of the base.

A heater provides a stable base temperature for thermal-vacuum applications. The heater consists of two independent thermostatically controlled heaters mounted in an aluminum cylinder the same diameter as the radiometer base. The heater, operating off 100 V (ac), controls the radiometer base at approximately 90° ± 3°F.

The Boelter-Schmidt-type sensors were selected for this radiometer because they are very rugged and measure heat transfer, regardless of mode (absorption, emission, conduction, convection). The versatility of the radiometer is due to its split-disc configuration and the data-reduction technique used to determine the heat transferred into the sensors by each mode, or a selected combination of modes.

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